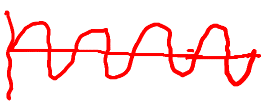
Unit-3 (Forced vibrations)

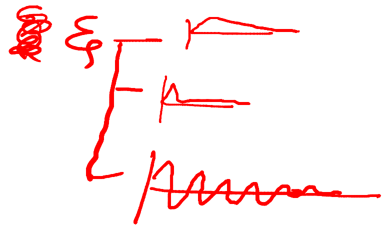
SDOF System.

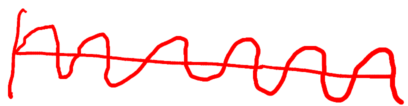
Vibration

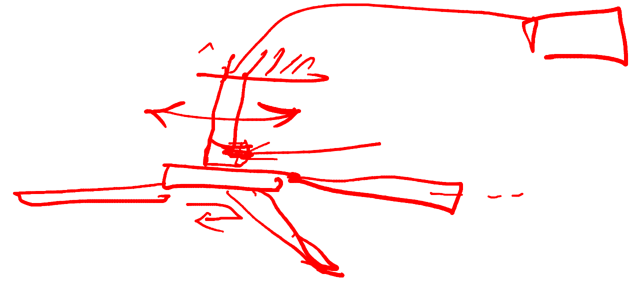
Initial displace → Free (II) | Forced vibration (III)

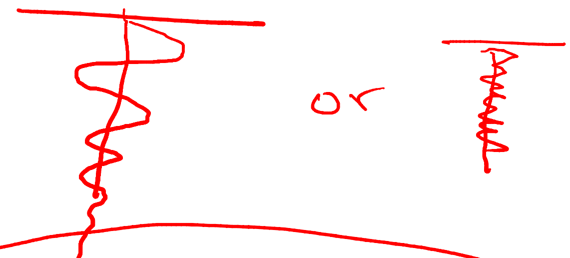
Free → Free Undamped vibration | Free damped vibration









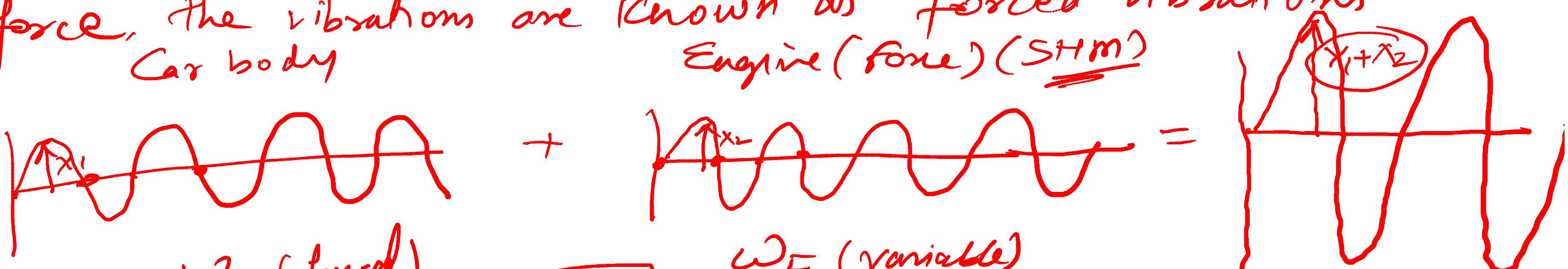


Cyclic force (Dynamic force)

If the system vibrates under the influence of external periodic force, the vibrations are known as forced vibrations

Car body

Engine (force) (SHM)



ω_n (fixed)

$\boxed{\omega_n = \omega_E}$

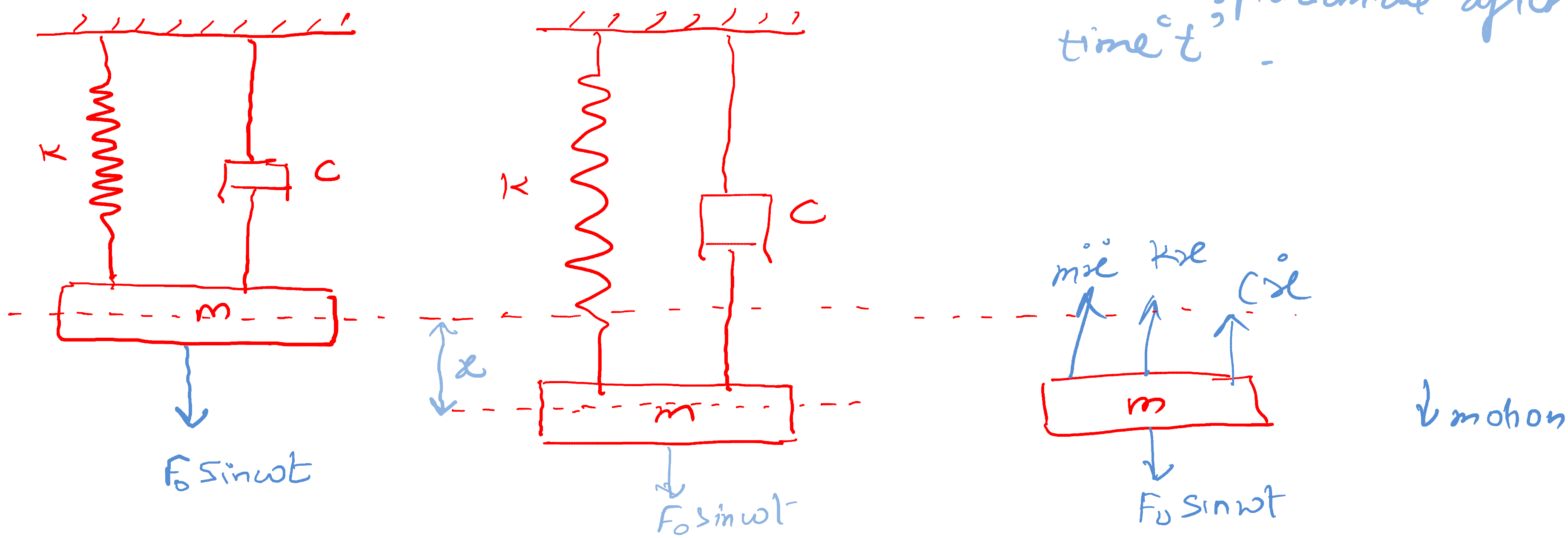
ω_E (variable)

Violent vibration

Resonance.

Forced damped vibrations with Constant Harmonic Excitation

x is displacement after time 't'.



By D'Alembert's Principle

Σ(External forces + Inertia forces) = 0

$$-F_0 \sin\omega t + kx + c\dot{x} + m\ddot{x} = 0$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t}$$

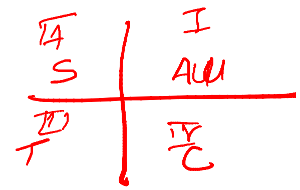
Linear Differential equation of 2nd order.

damped free

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = F_0 \sin\omega t}$$ — Linear second order Differential eqtn

Complimentry function (x_c) — Particular Integral (x_p)



$$x = x_c + x_p$$

1) Complimentry function :— (x_c)

$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\boxed{x_c = X_1 \cdot e^{-\xi\omega_n t} \cdot \sin(\omega_d t + \phi)}$$ ← underdamped free vibration

Solution derived in unit 2

2) Particular Integral (x_p) :—

$$\boxed{x_p = X \sin(\omega t - \phi)}$$ — (a)

~~1~~ Analytical method

2 Graphical — " —

$$\dot{x}_p = X\omega \cos(\omega t - \phi)$$

$$\boxed{\dot{x}_p = X\omega \sin\left(\omega t - \phi + \frac{\pi}{2}\right)}$$ — (b)

$$\frac{d\dot{x}_p}{dt} = \frac{d}{dt}[X\omega\cos(\omega t - \phi)] \Rightarrow -X\omega^2 \sin(\omega t - \phi)$$

$$\boxed{\ddot{x}_p = X\omega^2 \sin(\omega t - \phi + \pi)}$$ — (c)

$$m\ddot{x}_p + c\dot{x}_p + kx_p = F_0 \sin \omega t$$

$$m\left[\omega^2 x \sin(\omega t - \phi + \pi)\right] + c\left[\omega x \sin\left(\omega t - \phi + \frac{\pi}{2}\right)\right] + k\left[x \sin(\omega t - \phi)\right] = F_0 \sin \omega t .$$

$$-m\omega^2 x \sin(\omega t - \phi) + c\omega x \sin\left(\omega t - \phi + \frac{\pi}{2}\right) + kx \sin(\omega t - \phi) = F_0 \sin \omega t$$

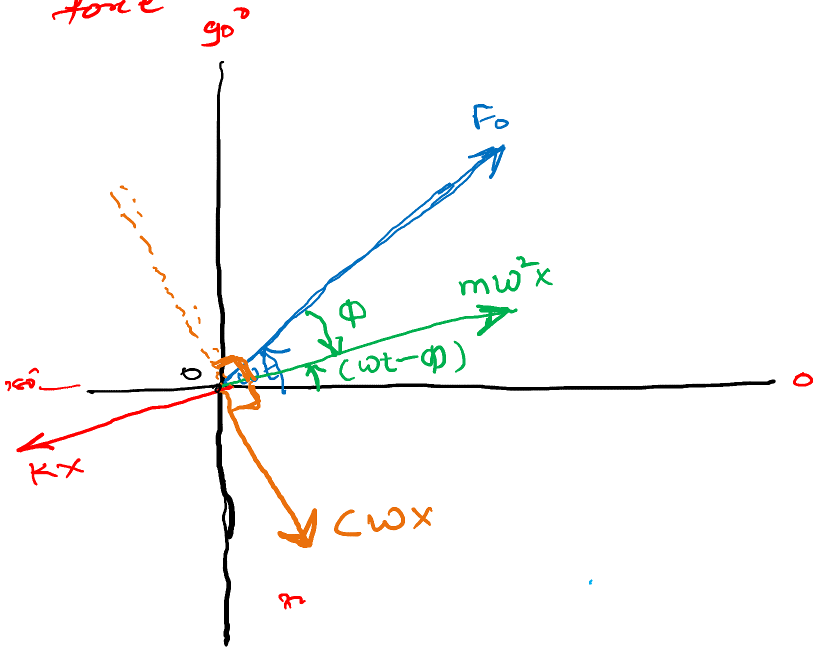
$$\boxed{\underbrace{F_0 \sin \omega t}_{①} + \underbrace{m\omega^2 x \sin(\omega t - \phi)}_{②} - \underbrace{c\omega x \sin\left(\omega t - \phi + \frac{\pi}{2}\right)}_{③} - \underbrace{kx \sin(\omega t - \phi)}_{④} = 0}$$

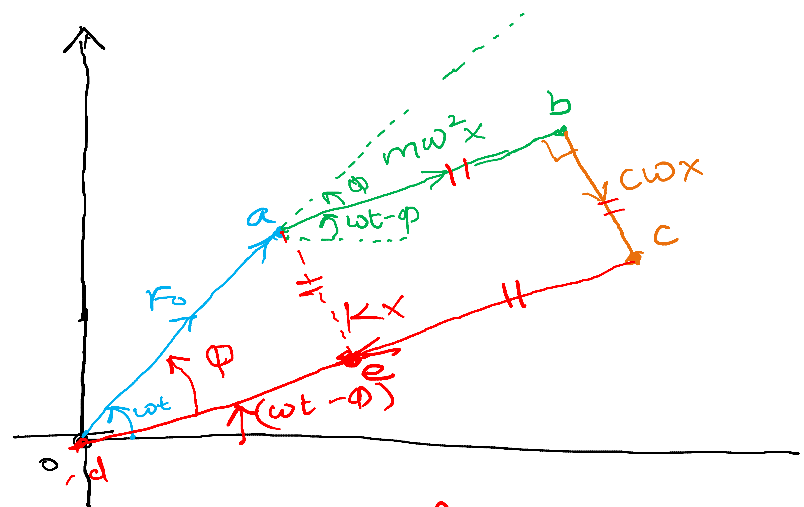
① harmonic force

② Inertia force term

③ Damping force

④ Spring force.





Force Polygon

$$\tan\phi = \frac{ae}{de} \Rightarrow \frac{bc}{oc - ec} \Rightarrow \frac{bc}{oc - ab}$$

$$\tan\phi = \frac{bc}{oc - ab}$$

$$= \frac{c\omega x}{kx - m\omega^2 x}$$

$$= \frac{c\omega}{k - m\omega^2}$$

$$\tan\phi = \frac{\left[c\omega/k\right]}{\left[1 - \frac{m\omega^2}{k}\right]}$$

$$\boxed{\tan\phi = \left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right]}$$

$$\frac{c\omega}{k} \Rightarrow \frac{c_c}{c_c} \cdot \frac{c \cdot \omega}{k}$$

$$\Rightarrow \frac{c}{c_c} \cdot \frac{c_c \cdot \omega}{k}$$

$$\Rightarrow \xi \cdot (2m\omega_n) \frac{\omega}{k}$$

$$\Rightarrow \xi \cdot \left[2m \cdot \sqrt{\frac{k}{m}}\right] \cdot \frac{\omega}{k}$$

$$\Rightarrow \xi\, 2\sqrt{k \cdot m} \cdot \frac{\omega}{k}$$

$$\Rightarrow \xi\, 2 \cdot \omega \cdot \sqrt{\frac{k \cdot m}{k^{\cancel{2}}}}$$

$$\boxed{\frac{c\omega}{k} \Rightarrow 2\xi \frac{\omega}{\omega_n}}$$

$$1 - \frac{m\omega^2}{k} \Rightarrow 1 - \left(\frac{m}{k}\right)\omega^2$$

$$\Rightarrow 1 - \frac{\omega^2}{\omega_n^2}$$

$$\boxed{1 - \frac{m\omega^2}{k} \Rightarrow 1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

By pythagoras theorem.

$$oa^2 = ae^2 + oe^2$$

$$oa = \sqrt{ae^2 + oe^2}$$

$$F_0 = \sqrt{(c\omega x)^2 + (Kx - m\omega^2 x)^2}$$

$$F_0 = X\sqrt{(c\omega)^2 + (K - m\omega^2)^2}$$

$$X = \frac{F_0}{\sqrt{(c\omega)^2 + (K - m\omega^2)^2}}$$

$$X = \frac{F_0/K}{\frac{1}{K}\left[\sqrt{(c\omega)^2 + (K - m\omega^2)^2}\right]}$$

$$X = \frac{F_0/K}{\sqrt{\left(\frac{c\omega}{K}\right)^2 + \left(1 - \frac{m\omega^2}{K}\right)^2}}$$

$$\boxed{X = \frac{F_0/K}{\sqrt{\left[2\xi\frac{\omega}{\omega_n}\right]^2 + \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2}}}$$

Amplitude.

Particular Integral.

$$x_p = X \sin(\omega t - \phi)$$

$$\boxed{x_p = \frac{F_0/K \cdot \sin(\omega t - \phi)}{\sqrt{\left(2\xi\frac{\omega}{\omega_n}\right)^2 + \left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2}}}$$

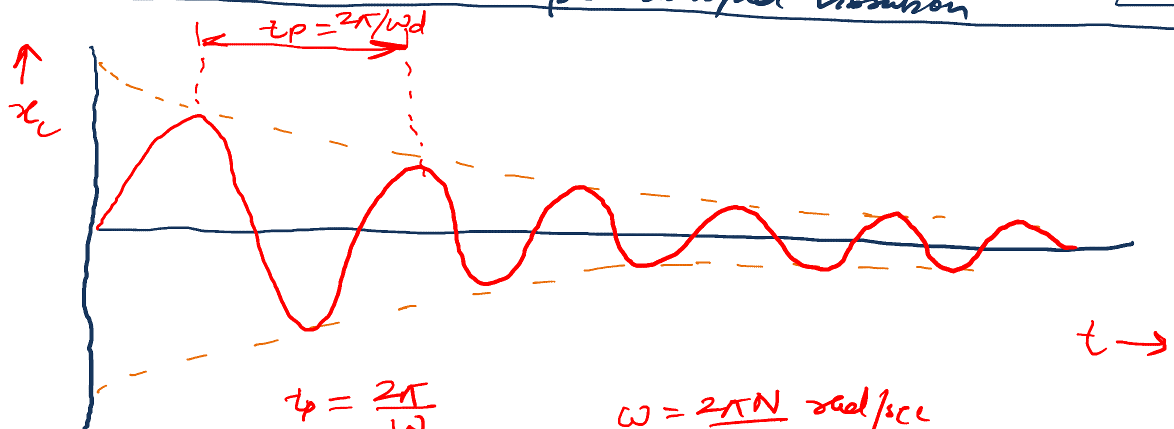
$$x = x_c + x_p$$

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t \quad \text{solution}$$

$$x = x_c + x_p$$

$$x = \underbrace{X_1 \cdot e^{-\xi_1 \omega_n t} \cdot \sin\left[\left(\sqrt{1-\xi_1^2}\right)\omega_n \cdot t + \phi_1\right]}_{x_c \text{ free damped vibration}} + \frac{F_0 \sin(\omega t - \phi)}{k \cdot \sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi_1\left(\frac{\omega}{\omega_n}\right)\right]^2}}$$

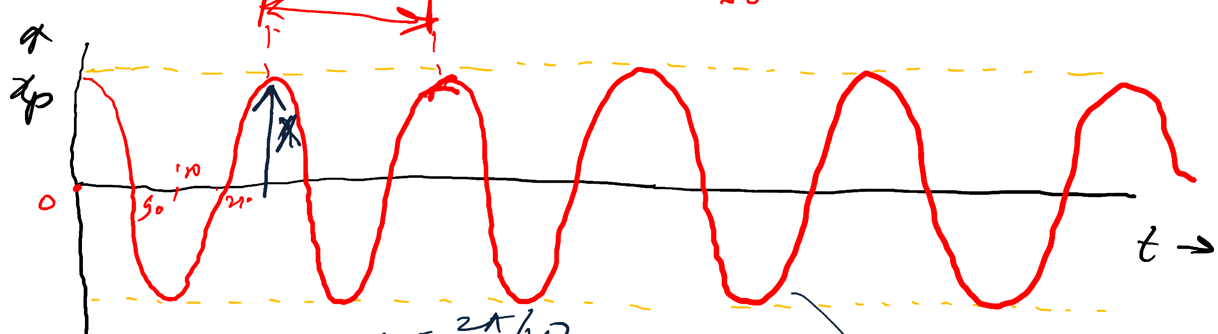
x_p ← External force
Motor/Engine/Fan



x_c ← (Transient vibration)
↓
decaying

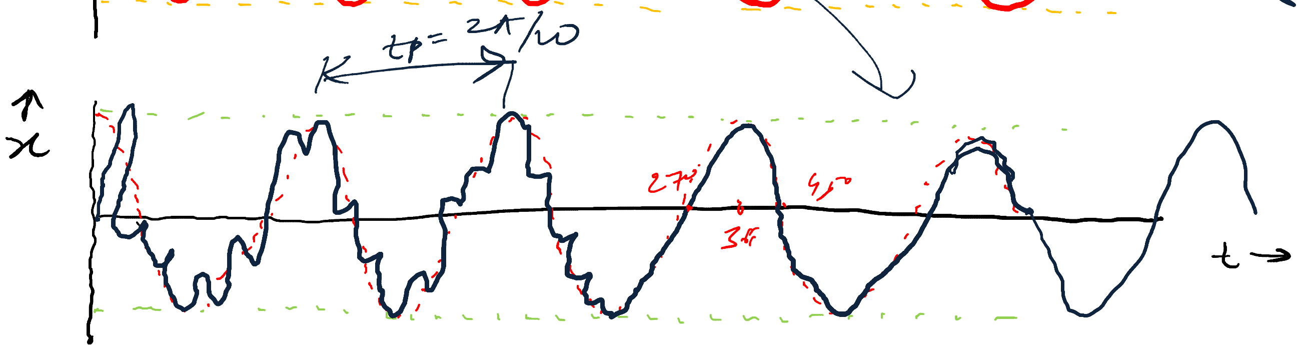
← vehicle body (chassis seats +)

$\omega = \frac{2\pi N}{60}$ rad/sec



← x_p (Steady state vibration)

← Engine



← x Complete solution

* Magnification factor :- (Amplification factor / Amplitude ratio / dynamic magnifier)

$$\boxed{M.F. = \frac{\text{Amplitude of steady state vibration}}{\text{zero frequency deflection (due to force } F_0)}} = \frac{X}{X_{st}}$$

$$M.F = \frac{\dfrac{\cancel{F_0/K}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}}{\cancel{F_0/K}}$$

$$X_{st} = \frac{F_0/K}{\sqrt{[1-0]+[0]}}$$

$$X_{st} = \frac{F_0/K}{1}$$

$$\boxed{X_{st} = F_0/K}$$

$$\boxed{M.F = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}} \; *$$

$$\boxed{X = X_{st}(M.F)} \; *$$

* Frequency Response Curves:-

Resonance is occurring $(\omega = \omega_n)$
(critical point)

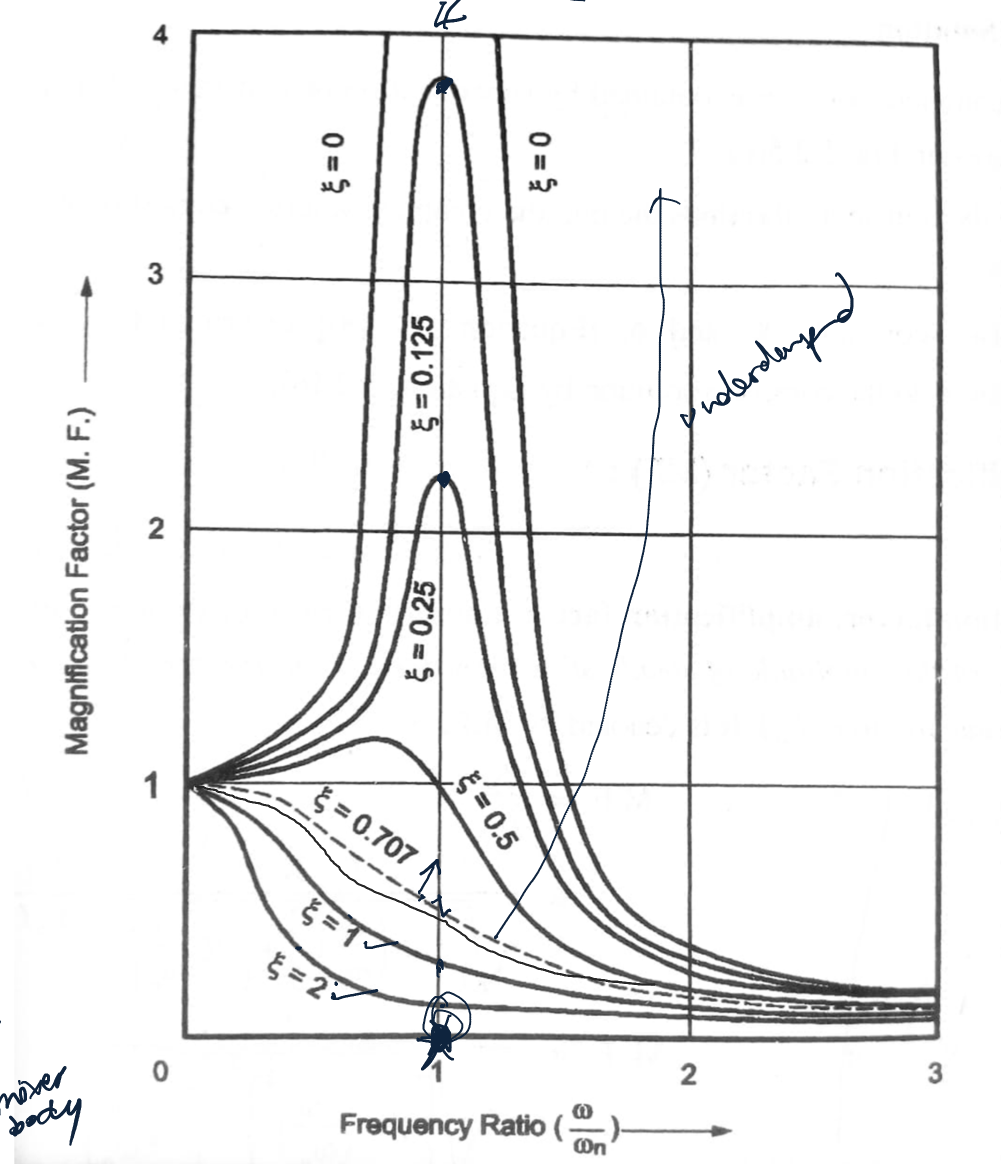
$$\xi = \frac{c}{c_c}$$

$\xi > 1$ — overdamped
$\xi = 1$ — critical
$\xi < 1$ — underdamped

$$M.F = \frac{X}{X_{st}}$$

↓ magnification ↑
↑ damping ↓

$\xi = 0.707$ ↓



mixer body
ω_n natural frequency

ω → frequency of external force (steady state vibration)
motor
$\frac{2\pi N}{60}$

$\frac{\omega}{\omega_n} > 1$

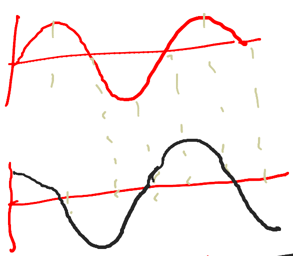
$\omega > \omega_n$

motor > mixer body

(2) Phase angle vs frequency ratio curve.

$\xi < 1$

Source vibration

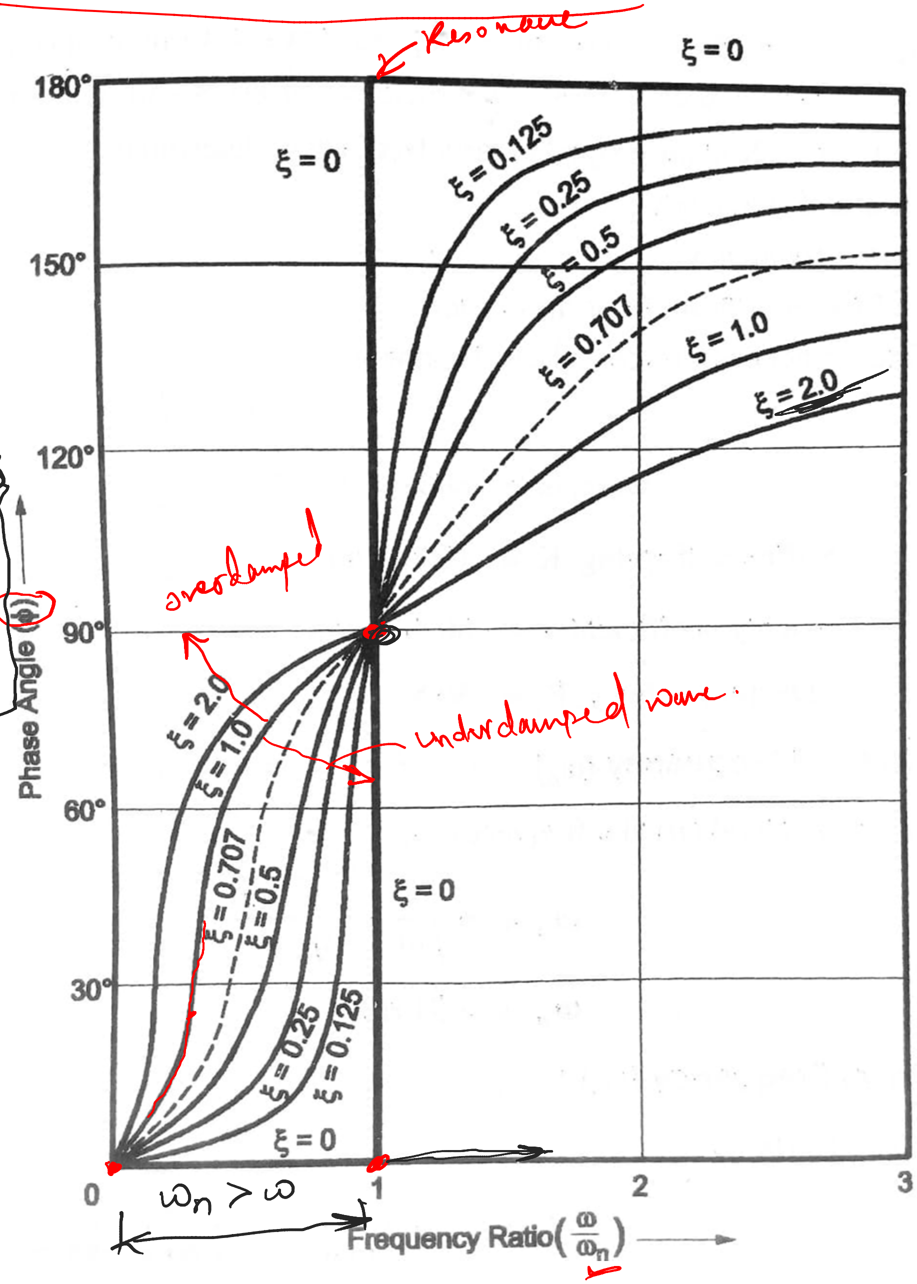


Actual body

$$\phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\frac{2\xi}{0}$$

(2)



$\omega \rightarrow$ frequency of damped vibration

$\omega_n =$ natural frequency of free vibration

$\boxed{\omega = \omega_n}$

Resonance condition

$\boxed{\frac{\omega}{\omega_n} = 1}$

$\boxed{\omega_n > \omega}$

$\xi \downarrow \quad \phi \downarrow$

$\frac{\omega}{\omega_n} > 1$

$\boxed{\omega > \omega_n}$

$\xi \downarrow \quad \phi \uparrow$

In a vibrating system, a mass of 3 kg is suspended by a spring of stiffness 1200 N/m and it is subjected to a harmonic excitation of 20 N. If the viscous damper is provided with the damping coefficient of 75 N-s/m, determine :

(1) the resonance frequency ;
(2) the phase angle at resonance ;
(3) the amplitude at resonance ;
(4) the frequency corresponding to peak amplitude ; and
(5) the damped frequency ;

PU - Dec. 2007

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

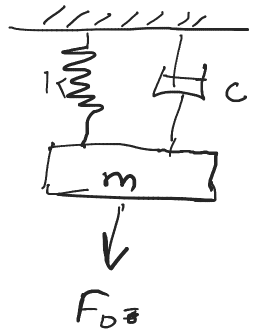
Ans Given data :-

$m = 3\,kg$

$K = 1200\,N/m$

$F_0 = 20\,N$

$C = 75\,N\text{-}s/m$



$$\omega_n = \sqrt{\frac{K}{m}}$$

$$= \sqrt{\frac{1200}{3}}$$

① $\boxed{\omega_n = 20\,rad/sec}$ ← resonance frequency.

② $$\phi = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$\omega = \omega_n$ — at resonance.

$$\phi = \tan^{-1}\left[\frac{2\xi}{1-(1)^2}\right]$$

$$\phi = \tan^{-1}\left[\frac{2\xi}{0}\right]$$

$$\phi = \tan^{-1}(\infty)$$

$\boxed{\phi = 90^\circ}$ → Phase angle at Resonance.

$$\xi = \frac{C}{C_c} \Rightarrow \frac{75}{2m\omega_n} \Rightarrow \frac{75}{2(3)(20)}$$

$$\boxed{\xi = 0.625}$$

③ Amplitude at Resonance

$$X = \frac{F_0/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

Resonance $\omega = \omega_n$ so, $\boxed{\frac{\omega}{\omega_n} = 1}$

$$X = \frac{F_0/K}{\sqrt{\left[1-(1)^2\right]^2 + \left[2\xi(1)\right]^2}}$$

$$X = \frac{F_0/K}{\sqrt{(2\xi)^2}} \Rightarrow \frac{F_0/K}{2\xi}$$

$$X = \frac{20/1200}{2(0.625)}$$

$$\boxed{X = 0.0133m}$$

④ Frequency corresponding to Peak amplitude.

$$X = \frac{X_{st}}{\sqrt{\left[1-\left(\frac{\omega_p}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega_p}{\omega_n}\right]^2}}$$

M.F

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega_p}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega_p}{\omega_n}\right]^2}}$$

$\frac{dX}{dX_{st}} = 0$ — maxima.

$\frac{\omega_p}{\omega_n} \rightarrow a$

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{\left[1-a^2\right]^2 + \left[2\xi a\right]^2}}$$

$$\frac{dX}{dX_{st}} = \frac{2(1-a^2)(-2a) + 2(2\xi \cdot a)(2\xi)}{2\left\{(1-a^2)^2 + (2\xi a)^2\right\}^{3/2}}$$

$$0 = \frac{-4a + 4a^3 + 8a\xi^2}{2\left[(1-a^2)^2 + (2\xi a)^2\right]^{3/2}}$$

$$0 = -1 + a^2 + 2\xi^2$$

$$\boxed{a = \sqrt{1-2\xi^2}} \qquad \frac{\omega_p}{\omega_n} = \sqrt{1-2\xi^2}$$

$$\frac{\omega_p}{\omega_n} = \sqrt{1-2\xi^2}$$

$$\omega_p = \omega_n\sqrt{1-2\xi^2}$$

$$\boxed{\omega_p = 9.35 \text{ rad/sec}}$$ ← force.

(5) damped frequency (ω_d)

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

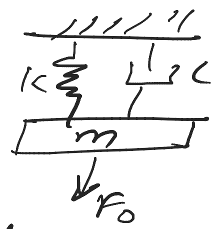
$$\boxed{\omega_d = 15.61 \text{ rad/sec.}}$$ ← without force

A machine part of a mass 2 kg vibrates in a viscous medium. Determine the damping co-efficient, when a harmonic exciting force of 25 N results in a resonant amplitude of 12.5 mm with a period of 0.20 sec. If the system is excited by a harmonic force of frequency 4 cycles/sec., determine the percentage increase in the amplitude of forced vibration, when the damper is removed. **PU - Dec. 2008**

Ans. $m = 2\,kg$

$c = ?$

$F_0 = 25\,N$ - harmonic force



$X = 12.5\,mm$ - Resonant amplitude
$= 0.0125\,m$

$\omega_n = \omega$

$t_p = 0.2\,sec$

$f = 4$ cycles/sec or Hz

Circular frequency of machine :

$\omega = \dfrac{2\pi}{t_p}$ ← Resonant freq -

$\omega = \dfrac{2\pi}{0.2}$

$\boxed{\omega = 31.416\ rad/sec} = \omega_n$

$\omega_n = \sqrt{\dfrac{k}{m}}$

$31.416 = \sqrt{\dfrac{k}{2}}$

$\boxed{k = 1973.93\ N/m}$

$$\boxed{X = \frac{F_0/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}}$$

$$X = \frac{F_0/k}{\sqrt{\left[1-(1)^2\right]^2 + \left[2\xi(1)\right]^2}}$$

$$X = \frac{F_0/k}{2\xi}$$

$$0.0125 = \frac{25/1973.93}{2\xi}$$

$$\boxed{\xi = 0.506}$$

$$\xi = \frac{c}{C_c}$$

$$\xi = \frac{c}{2m\omega_n}$$

$$\boxed{c = 63.66\ N\text{-}s/m}$$

$$x' = \frac{F_o/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2}}$$

$$x' = \frac{F_o/k}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$x' = \frac{25/1973.93}{1-\left(\frac{25.13}{31.416}\right)^2}$$

$f = 4$ cycles/sec or Hz

$$\omega = 2\pi \cdot f$$

$$\omega = (2\pi)(4)$$

$$\boxed{\omega = 25.13\ rad/sec}$$

$$\boxed{x' = 0.0351m}$$

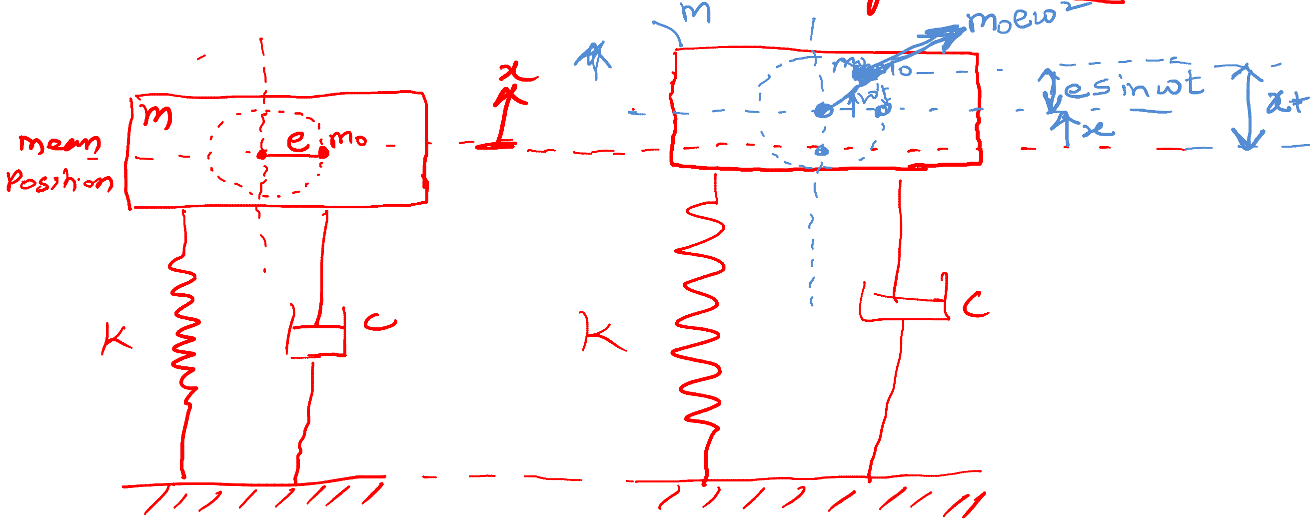
$$x'' = \frac{F_o/k}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right)^2}}$$

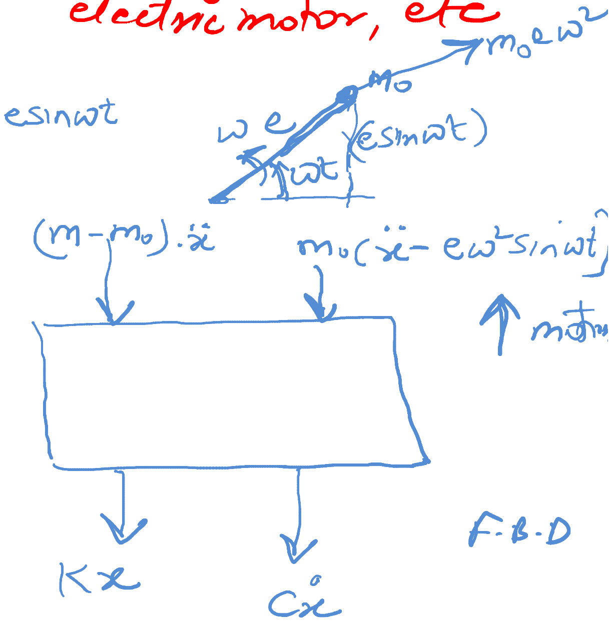
$$x'' = \frac{25/1973.93}{\sqrt{\left(1-\frac{25.13}{31.41}\right)^2 + \left[2(0.\ )\left(\frac{25.13}{31.41}\right)\right]^2}}$$

$$\boxed{x'' = 0.0143m}$$

$$\frac{x'-x''}{x''} = 145.45\%$$

* Forced vibrations due to rotating unbalance :- e.g → centrifugal pump, turbines, electric motor, etc





Inertia force of m_0.

$$= m_0 \frac{d^2}{dt^2}(x + e\sin\omega t)$$

$$= m_0 \frac{d}{dt}\left[\dot{x} + e\omega\cos\omega t\right]$$

$$= m_0\left[\ddot{x} - e\omega^2\sin\omega t\right]$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = F_0\sin\omega t}$$

$$\boxed{F_0 = m_0 e\omega^2}$$

By D'Alembert's Principle

$$(m-m_0)\ddot{x} + m_0(\ddot{x} - e\omega^2\sin\omega t) + kx + c\dot{x} = 0$$

$$m\ddot{x} - \cancel{m_0\ddot{x}} + \cancel{m_0\ddot{x}} - m_0 e\omega^2\sin\omega t + kx + c\dot{x} = 0$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = m_0 e\omega^2\sin\omega t}$$

Fundamental eqn for forced vibration due to rotating unbalance

$$\boxed{x = X_1 e^{-\xi \omega_n t} \sin\left[\sqrt{1-\xi^2}\,\omega_n t + \phi_1\right] + \frac{(m_0 e \omega^2) \cdot \sin(\omega t - \phi)}{K\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}}$$

Amplitude

$$X = \frac{m_0 \cdot e\,\omega^2 / K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\omega_n = \sqrt{\frac{K}{m}}$$

$$\boxed{m \cdot \omega_n^2 = K}$$

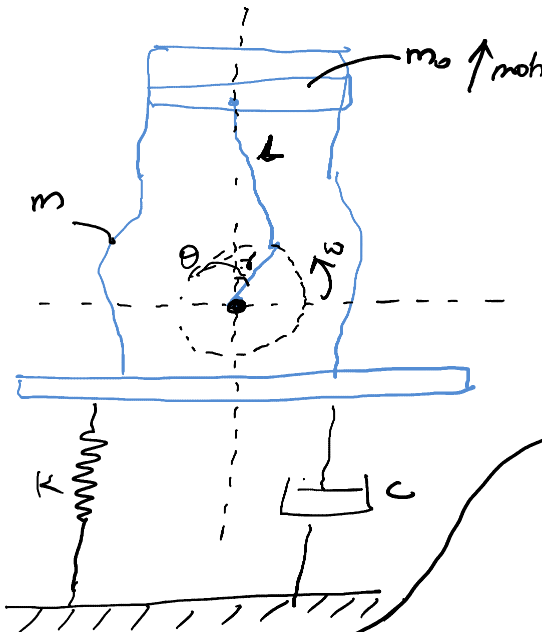
$$X = \frac{m_0 \cdot e\,\omega^2 / m \cdot \omega_n^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\boxed{\frac{X}{\left(\frac{m_0 \cdot e}{m}\right)} = \frac{\left(\omega/\omega_n\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}}$$

Phase difference:-

$$\boxed{\phi = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]}$$

* Forced vibrations due to Reciprocating unbalanced: (e.g. I.C. Engine, reciprocating compr., rec. pumps.



*Unbalanced in reciprocating masses is due to the Inertia force of Piston

$$\boxed{F_I = m_0 \cdot r \cdot \omega^2 \left[\sin\omega t + \frac{\sin 2\omega t}{n} \right]}$$

m_0 = mass of Piston

r = radius of crank

ω = Ang. speed of crank

n = obliquity ratio = $\frac{l}{r}$

So as the value of $\frac{\sin 2\omega t}{n}$ is very small, we can neglect it

$$\boxed{F_I = m_0 \cdot r \cdot \omega^2 [\sin\omega t]}$$

$$\boxed{F_0 = F_I = m_0 \cdot r \cdot \omega^2 \sin\omega t}$$

Qu. A system having rotating unbalanced has total mass of 25 kg The unbalance mass of 1 kg rotates with a radius of 0.04 m. It has been observed that at a speed of 1000 rpm, The system and the eccentric mass has a phase difference of $90°$ and the corresponding amplitude is 0.015 m. find

① $\omega_n = ?$ of the system

② Damping factor $(\xi) = ?$

③ Amplitude at 1500 rpm

④ Phase angle at —u—.

Ans

$m = 25$ kg

$m_0 = 1$ kg

$e = 0.04$ m

$N = 1000$ rpm

$\omega = \frac{2\pi N}{60} = 104.71$ rad/sec

$\phi = 90° \rightarrow$ resonance $\omega = \omega_n$

$X = 0.015$ m.

$$\phi = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$90° = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\infty = \frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$\frac{1}{0} = \frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}$$

$$1-\left(\frac{\omega}{\omega_n}\right)^2 = 0$$

$$\left(\frac{\omega}{\omega_n}\right)^2 = 1$$

$$\frac{\omega}{\omega_n} = 1$$

$$\omega = \omega_n$$

$$\boxed{\omega_n = 104.71 \text{ rad/sec}}$$

$$\frac{X}{\left(\frac{m_o \cdot e}{m}\right)} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\frac{X}{\left(\frac{m_o \cdot e}{m}\right)} = \frac{(1)^2}{\sqrt{\left[1-(1)^2\right]^2 + \left[2\xi(1)\right]^2}}$$

$$\frac{X}{\left(\frac{m_o \cdot e}{m}\right)} = \frac{1}{2\xi}$$

$$\xi = \frac{m_o \cdot e/m}{2X}$$

$$\xi = \frac{(1)(0.04)}{(2)(25)(0.015)}$$

$$\boxed{\xi = 0.0533}$$

$N = 1500$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(1500)}{60}$$

$$\boxed{\omega = 157.07 \text{ rad/sec}}$$

$$\frac{X}{\left[\frac{(1)(0.04)}{25}\right]} = \frac{\left[\frac{157.07}{104.71}\right]^2}{\sqrt{\left[1-\left(\frac{157.07}{104.71}\right)^2\right]^2 + \left[2(0.0533)\left(\frac{157.07}{104.71}\right)\right]^2}}$$

$$\boxed{X = 0.001791 \text{ m}}$$

or

$$\boxed{X = 1.79 \text{ mm}}$$

$$\phi = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\phi = \tan^{-1}\left[-0.12792\right]$$

$$\boxed{\phi = 172.71^\circ}$$ or $\underline{352.71^\circ}$

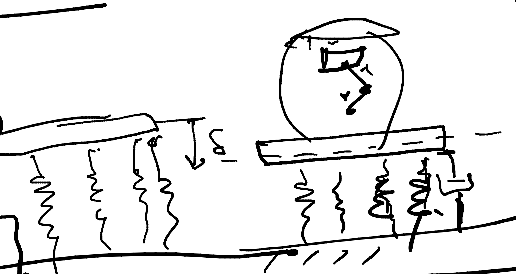
Qu: When a single cylinder Engine of total mass 300 kg is placed on 4 springs is compressed by 2 mm. A dashpot, offering 400 N of damping force at a relative velocity of 200 mm/sec. is attached to the engine to damp out the vibrations. The reciprocating mass of the engine is 20 kg and stroke of piston is 130 mm. If the engine is running at 1500 rpm. neglecting secondary force find the amplitude of steady state vibrations.

$$F = \frac{mg}{4} = \frac{300 \times 9.81}{4}$$

$$\delta = 0.002\,m$$

$$K = \frac{F}{\delta} = \frac{300(9.81)}{4(0.002)}$$

$$\boxed{K = 0.36 \times 10^{6}\ N/m}$$



Ans Given:-

$$\boxed{m = 300\,kg}$$

$$\delta = 2\,mm = 0.002\,m$$

dashpot

$F = 400\,N$

$V = \dot{x} = 200\,mm/sec = 0.2\,m/sec$

$$c = \frac{F}{\dot{x}} = \frac{400}{0.2} = 2000\ N\text{-}sec/m$$

$$\boxed{m_0 = 20\,kg}$$

Stroke = 130 mm = 0.13 m = 2r

$$\boxed{r = 0.065\,m}$$

$N = 1500$ rpm

$$\omega = \frac{2\pi N}{60} = 157.058\ rad/sec$$

$$X = \frac{\left(\frac{m_0 \cdot r}{m}\right)\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta \frac{\omega}{\omega_n}\right]^2}}$$

$$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{4K}{300}}$$

$$\omega_n = \sqrt{\frac{4(0.36 \times 10^{6})}{300}} = 70.03\ rad/sec$$

$$\xi = \frac{c}{c_c}$$

$$= \frac{c}{2 m \omega_n}$$

$$= \frac{2000}{2(300)(70.03)}$$

$$\boxed{\xi = 0.0476}$$

$$x = 0.00540 m$$

06

$$\boxed{x = 5.40 mm}$$

Qn A Single cylinder vertical petrol engine of total mass 400kg is mounted upon a steel chasis frame and causes a vertical static deflection of 4mm. The reciprocating parts of engine have a mass of 20kg and move through a vertical stroke of 150mm with approximately S.H.M. If the dashpot is provided with damping resistance of 1600 N-sec/m. determine

① Speed of driving shaft at which resonance will occur

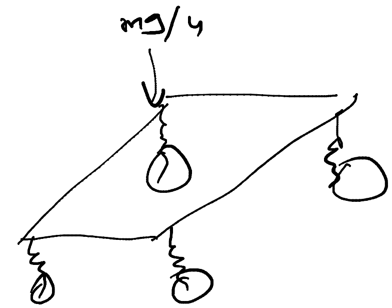
② Amplitude of steady state vibration when the driving shaft of engine rotate at 800 rpm.

An Given:-

$m = 400kg$

$\delta = 4mm$

H.W



$k = \dfrac{\frac{mg}{4}}{\delta} = \dfrac{mg}{4\delta} =$

$k_e = 4k =$

$\omega_n = \sqrt{\dfrac{k_e}{m}}$

$N = \dfrac{\omega_n \cdot 60}{2\pi}$

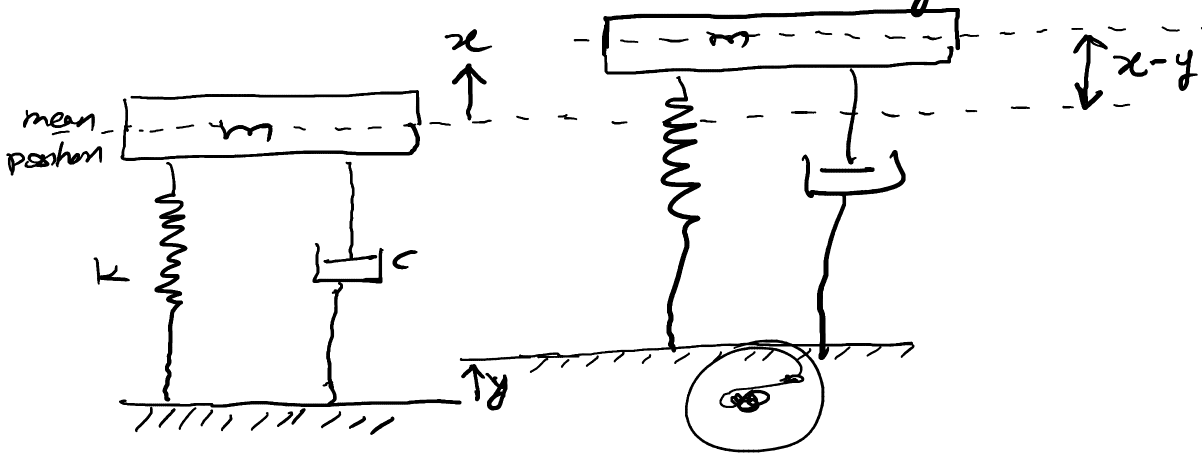
$\boxed{N = 471.63\ rpm}$

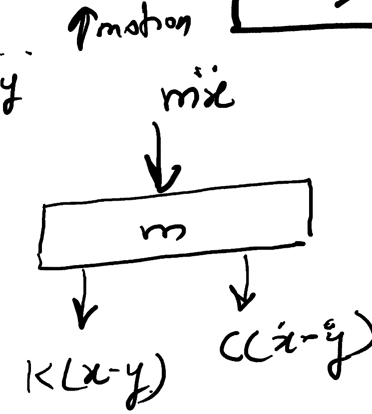
resonance speed

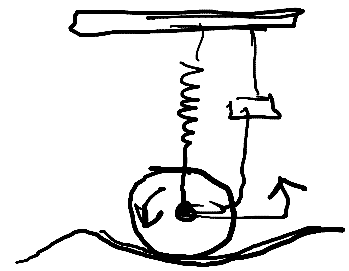
$\boxed{X = 7.67 \times 10^{-3} m.}$

* **Base excitation** or [Force vibrations due to Excitation of support instead of mass]

$\boxed{x > y}$







$$\boxed{y = Y \cdot \sin\omega t}$$

$$\dot{y} = Y\omega \cdot \cos\omega t$$

Absolute amplitude method:

$$m\ddot{x} + k(x-y) + c(\dot{x}-\dot{y}) = 0$$

$$m\ddot{x} + c\dot{x} - c\dot{y} + kx - ky = 0$$

$$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$m\ddot{x} + c\dot{x} + kx = c[Y\omega \cdot \cos\omega t] + k[Y\sin\omega t]$$

$$= Y[k\sin\omega t + c\omega\cos\omega t] \cdot \frac{\sqrt{k^2+c^2\omega^2}}{\sqrt{k^2+c^2\omega^2}}$$

$$=$$

$$m\ddot{x} + c\dot{x} + kx = Y \cdot \sqrt{k^2 + c^2\omega^2} \left[\frac{k}{\sqrt{k^2 + c^2\omega^2}} \cdot \sin \omega t + \frac{c\omega}{\sqrt{k^2 + c^2\omega^2}} \cos \omega t \right]$$

assume, let

$$\boxed{\frac{k}{\sqrt{k^2 + c^2\omega^2}} = \cos \alpha}$$

$$\boxed{\frac{c\omega}{\sqrt{k^2 + c^2\omega^2}} = \sin \alpha}$$

$$m\ddot{x} + c\dot{x} + kx = Y \cdot \sqrt{k^2 + c^2\omega^2} \left[\cos \alpha \cdot \sin \omega t + \sin \alpha \cdot \cos \omega t \right]$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = \underbrace{Y \cdot \sqrt{k^2 + c^2\omega^2}}_{F_0} \sin(\omega t + \alpha)}$$

– Absolute differential equation of motion

$$F_0 = Y \cdot \sqrt{k^2 + c^2\omega^2}$$

$$\boxed{m\ddot{x} + c\dot{x} + kx = F_0 \sin(\omega t + \alpha)}$$

$$\boxed{x = X_1 \cdot e^{-\xi \omega_n t} \sin(\omega_d t + \phi) + X \sin[\omega t - (\phi - \alpha)]}$$

Steady state amplitude eqn

$$X = \frac{Y\sqrt{K^2+c^2\omega^2}/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

$$\frac{\left(\frac{1}{\cancel{K}}\sqrt{1+\frac{c^2\omega^2}{K^2}}\right)}{\cancel{K}}$$

$$\sqrt{1+\left(2\xi\left(\frac{\omega}{\omega_n}\right)\right)^2}$$

$$\boxed{\frac{X}{Y} = \frac{\sqrt{K^2+c^2\omega^2}/K}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}}$$

1. when $\omega << \omega_n$ $\frac{X}{Y} \simeq 1$ — rigid body.
2. when $\omega >> \omega_n$ $\frac{X}{Y} \simeq 0$ — ~~station~~ the mass will be stationary.
3. $\frac{\omega}{\omega_n} = \sqrt{2}$ $\frac{X}{Y} = 1$ — for all ξ values.

$$\boxed{\frac{X}{Y} = \frac{\sqrt{1+\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]^2}}{\sqrt{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right]+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}}$$

Phase angle equation :-

$$\phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

$$\alpha = \tan^{-1}\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]$$

$$\boxed{(\phi-\alpha) = \tan^{-1}\left[\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right] - \tan^{-1}\left[2\xi\left(\frac{\omega}{\omega_n}\right)\right]}$$

Phase angle formula
for Base excitation

$$\cos\alpha = \frac{K}{\sqrt{K^2+c^2\omega^2}}$$

$$\sin\alpha = \frac{c\omega}{\sqrt{K^2+c^2\omega^2}}$$

$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$= \frac{c\omega}{\cancel{\sqrt{K^2+c^2\omega^2}}} \times \frac{\cancel{\sqrt{K^2+c^2\omega^2}}}{K}$$

$$\boxed{\tan\alpha = \frac{c\omega}{K}}$$

or

$$\boxed{\tan\alpha = 2\xi\left(\frac{\omega}{\omega_n}\right)}$$

The suspension system of a vehicle has a spring constant of 500 kN/m and a damping ratio of 0.5. The vehicle has a speed of 80 km/hr. The road surface varies sinusoidally with an amplitude of 10 cm and a wavelength of 5 m. If the mass of a vehicle is 1000 kg, determine its amplitude of oscillations.

PU - May 2004

Given:-

$K = 500\,kN/m = 500\times10^{3}\,N/m$

$\xi = 0.5$

$V = 80\,km/hr$

$V = \dfrac{80\times10^{3}}{60\times60} = 22.22\,m/sec$

$Y = 10cm = 0.1m$

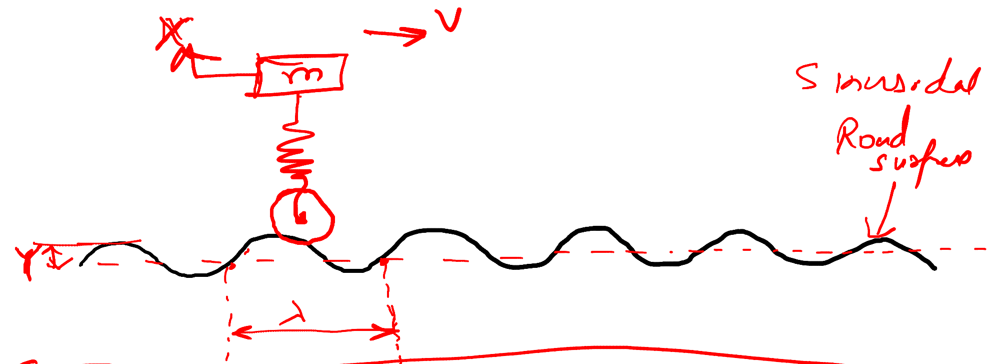
$\lambda = 5m$

$m = 1000kg$

$X = ?$

$\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{500\times10^{3}}{1000}}$

$$\boxed{\omega_n = 22.36\ rad/sec}$$



$$\boxed{\frac{X}{Y} = \frac{\sqrt{1+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left[2\xi\frac{\omega}{\omega_n}\right]^2}}}$$

$\lambda = \dfrac{\text{Velocity}}{\text{frequency}}$

$\lambda = V\cdot t_p$

$$\boxed{t_p = \frac{\lambda}{V}}$$

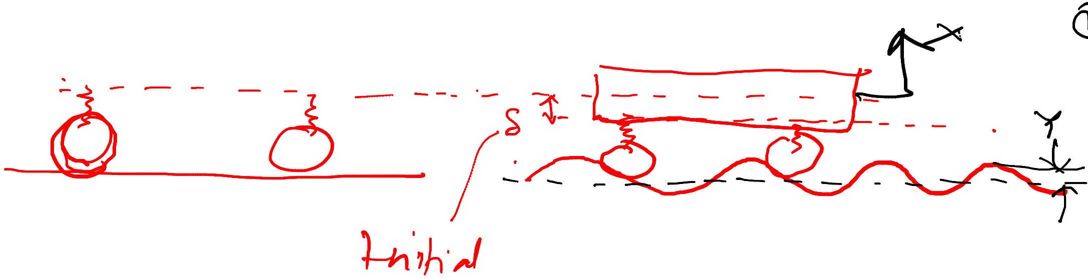
$t_p = \dfrac{5}{22.22} = 0.225\ sec$

$\omega = \dfrac{2\pi}{t_p} = \dfrac{2\pi}{0.225}$

$$\boxed{\omega = 27.92\ rad/sec}$$

$$\boxed{X = 0.11\,m}$$

The springs of an automobile trailer are compressed 0.1 m under its own weight. Find the critical speed when the trailer is travelling over a road with a profile approximated by a sine wave of amplitude 0.08 m and wavelength of 14 m. What will be the amplitude of vibration at 60 km/hour ?



① $\delta = 0.1\,m$

$V = ?$ — ⓐ Resonance condtn

$\boxed{\omega = \omega_n}$

$Y = 0.08\,m$

$\lambda = 14\,m$

② $X = ?$ ⓐ $V = 60\ km/hr.$

Critical speed of any system is that speed where the body is vibrating at its natural frequency (ω_n).

or

Resonance Condition: $\varphi = 0$

$$\boxed{t_p = \frac{\lambda}{V}}$$

$$\frac{2\pi}{\omega_n} = \frac{14}{V}$$

$$\frac{2\pi}{9.9} = \frac{14}{V}$$

22.05 km/hr

$$\boxed{V = 79.41\ km/hr}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\boxed{mg = k\delta}$$

$$\boxed{\frac{g}{\delta} = \frac{k}{m}}$$

$$\omega_n = \sqrt{\frac{g}{\delta}}$$

$$= \sqrt{\frac{9.81}{0.1}}$$

$$\boxed{\omega_n = 9.90\ rad/sec}$$

$$\frac{X}{Y} = \frac{\sqrt{1 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\zeta\frac{\omega}{\omega_n}\right]^2}}$$

$$\frac{X}{Y} = \frac{1}{1 - \left(\frac{\omega}{\omega_n}\right)^2}$$

$$\boxed{X = 105.76\,mm}$$

$V = 60\ km/hr$

$V = 16.66\ m/sec$

$$\frac{2\pi}{\omega} = \frac{\lambda}{V}$$

$$\boxed{\omega = }$$

Quality factor

$$M.F = \frac{X}{X_{st}} = \frac{1}{\sqrt{\left[1-\frac{\omega}{\omega_n}\right]^2 + \left[2\xi\frac{\omega}{\omega_n}\right]^2}}$$

The amplitude ratio (X/X_{st}) of a system at resonance (i.e $\omega = \omega_n$) is called Quality factor.

$$\frac{X}{X_{st}} = \frac{1}{\sqrt{[1-1]^2 + [2\xi]^2}}$$

$$\boxed{\frac{X}{X_{st}} = \frac{1}{2\xi}}$$ — Quality factor.

↓

It is also one half of the reciprocal of damping ratio

$(\xi < 0.05)$ light damping.